OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
ft or √	Follow through

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awa marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark schem assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work mu the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the wo looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, awa the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involv your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candida intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be give associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or e. n th of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correc ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the schem otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a par dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On th two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously inc Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accura given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in t rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks wil through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates oth are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) sh penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark schem contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appe (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question rema according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generall this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	
1	(i)		Either Attempt use of quotient rule	M1	allow numerator wrong way round m and both terms in numerator involvin for M1 condone minor errors such as abs absence of brackets, …
			Obtain $\dfrac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless sub indicates their 'presence'
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$
				[3]	
			Or Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule
			Obtain $3(2x+1)^{-1}-6x(2x+1)^{-2}$ or equiv	A1	
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv
1	(ii)		Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or
			Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
				[3]	
2	(i)		Either Attempt to find exact value of $\sin A$	M1	using right-angled triangle or identity or …
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M
				[2]	
			Or Attempt use of identity $1+\cot^2 A=\operatorname{cosec}^2 A$	M1	using $\cot A=\frac{1}{2}$; allow sign error in attempt at i
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M
2	(ii)		State or imply $\dfrac{2+\tan B}{1-2\tan B}=3$	B1	
			Attempt solution of equation of form $\dfrac{\text{linear in } t}{\text{linear in } t}=3$	M1	by sound process at least as far as $k\tan B=c$
			Obtain $\tan B=\frac{1}{7}$	A1	answer must be exact; ignore subsequent attem
				[3]	

Question			Answer	Marks	
3	(a)		Substitute $t=3$ in $\lvert 2t-1\rvert$ and obtain value 5	B1	not awarded for final $\lvert 5\rvert$ nor for $\pm$
			Substitute $t=-3$ in $\lvert 2t-1\rvert$ and apply modulus correctly to any negative value to obtain a positive value	M1	with no modulus signs remaining
			Obtain value 7 as final answer	A1	not awarded for final $\lvert 7\rvert$ nor for ± 7 NB: substitutions in $\lvert 2t+1\rvert$ will give 5 and 7 – a further step to $5<t<7$ – B1 M1 A0; answers $\pm 5, \pm 7$ – this is B0 M0 A0
				[3]	
3	(b)		Either Attempt solution of linear equation or inequality with signs of x different	M1	
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)
			Or 1 Attempt to square both sides	M1	obtaining at least 3 terms on each side
			Obtain $x^2-2\sqrt{2}x+2>x^2+6\sqrt{2}x+18$	A1	or equiv; or equation; condone $>$ here
			Or 2 Attempt sketches of $y=\lvert x-\sqrt{2}\rvert$, $y=\lvert x+3\sqrt{2}\rvert$	M1	
			Obtain $x=-\sqrt{2}$ at point of intersection	A1	or equiv
			Conclude with inequality of one of the following types: $x<k\sqrt{2},\quad x>k\sqrt{2},\quad x<\frac{k}{\sqrt{2}},\quad x>\frac{k}{\sqrt{2}}$	M1	any integer k
			Obtain $x<-\sqrt{2}$ or $-\sqrt{2}>x$ as final answer	A1 [4]	final answer $x<-\frac{2}{\sqrt{2}}$ (or similar unsimplified v

Question			Answer	Marks	
4	(i)		Attempt process involving logarithm to solve $e^{0.021t} = 2$	M1	with t the only variable; at least as 2 1
			Obtain 33	A1	or greater accuracy; ignore absence of, $\frac{\ln 2}{0.021}$ is A0
			State (or calculate separately to obtain) 99	B1√ [3]	following previous answer; no need to includ
4	(ii)		Differentiate to obtain $ke^{0.021t}$	M1	where k is any constant not equal to 250
			Obtain $250 \times 0.021\, e^{0.021t}$	A1	or simplified equiv $5.25e^{0.021t}$
			Substitute to obtain 8.4 or $\frac{42}{5}$	A1 [3]	or value rounding to 8.4 with no obvious error
5	(i)		Integrate to obtain form $k(3x+1)^{\frac{1}{2}}$	*M1	any non-zero constant k
			Obtain $4(3x+1)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; or $4u^{\frac{1}{2}}$ following subs
			Apply the limits and subtract the right way round	M1	dep *M
			Obtain $4\sqrt{28} - 4\sqrt{7}$ and show at least one intermediate step in confirming $4\sqrt{7}$	A1 [4]	AG; necessary detail required; decimal verifica $[\ldots]_2^9 = 4\sqrt{28} - 4\sqrt{7} = 4\sqrt{7}$ is A0; $[\ldots]_2^9 = 8\sqrt{}$
5	(ii)		State or imply volume is $\int \pi \left(\frac{6}{\sqrt{3x+1}}\right)^2 dx$ or equiv	B1	merely stating $\int \pi y^2\, dx$ not enough; condone for limits yet; π may be implied by its later ap
			Integrate to obtain $k\ln(3x+1)$	M1	any non-zero constant with or without π
			Obtain $12\pi\ln(3x+1)$ or $12\ln(3x+1)$	A1	or unsimplified equiv
			Substitute limits correct way round and show each logarithm property correctly applied	M1	allowing correct applications to incorrect result natural logarithm involved; evidence of $\ln 28 -$ M0
			Obtain $24\pi\ln 2$	A1 [5]	no need for explicit statement of value of k

Question			Answer	Marks	
6	(i)		Sketch more or less correct $y = \ln x$	B1	existing for positive and negative y … l to any scales given on axes; condone gra… crosses y-axis
			Sketch more or less correct $y = 8 - 2x^2$	B1	(roughly) symmetrical about y-axis; extend… quadrants for which $y < 0$; no need to indicat… each curve separately
			Indicate intersection by some mark on diagram (just a 'blob' sufficient) of by statement in words away from diagram	B1 **[3]**	needs each curve to be (more or less) correct in on curves being related to each other correctly
6	(ii)		Refer, in some way, to graphs crossing x-axis at $x = 1$ and $x = 2$ and that intersection is between these values	B1 **[1]**	AG; the values 1 and 2 may be assumed from p there; dependent on curves being (more or less) quadrant; carrying out the sign-change routine
6	(iii)		Obtain correct first iterate	B1	to at least 3 dp (except in the case of starting va
			Show correct iterative process	M1	involving at least 3 iterates in all; may be impl converging values
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates given to may be rounded or truncated
			Conclude with 1.917	A1 **[4]**	answer required to exactly 3 dp; answer only w process is 0/4
			$1 \to 2 \to 1.91139 \to 1.91731... \to 1.91690... \to 1.91693...$ $1.5 \to 1.94865... \to 1.91479... \to 1.91707... \to 1.91692...$ $2 \to 1.91139... \to 1.91731... \to 1.91690... \to 1.91693...$		
6	(iv)		Obtain 3.92 or greater accuracy Attempt $4 \times \ln(\text{part (iii) answer})$ Obtain y-coordinate 2.60	B1√ M1 A1 **[3]**	following their answer to part (iii) value required to exactly 2 dp (so A0 for 2.6 ar

Question			Answer	Marks	
7	(i)		Attempt use of product rule	M1	to produce expression of form (something non-zero) $\ln(2y+3)$ + lin
			Obtain $\ln(2y+3)$ …	A1	their derivative with brackets included
			Obtain … + $\dfrac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
				[3]	
7	(ii)		Substitute $y=0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'corr from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer
			Attempt to find value of y for which $x=0$	M1	allowing process leading only to $y=-4$
			Substitute $y=-1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value followin
				[5]	
8	(i)		Attempt completion of square at least as far as $(x+2a)^2$ or differentiation to find stationary point at least as far as linear equation involving two terms	*M1	or equiv but a must be present
			Obtain $(x+2a)^2-3a^2$ or $(-2a,-3a^2)$	A1	
			Attempt inequality involving appropriate y-value	M1	dep *M; allow <, > or ≤ here; allow use of x; c
			State $y \geq -3a^2$ or $f(x) \geq -3a^2$	A1	now with ≥ ; here $x \geq -3a^2$ is A0
				[4]	

Question			Answer	Marks	
8	(ii)		Attempt composition of f and g the right way round	*M1	algebraic or (part) numerical; need – once
			Obtain or imply $16x^2 - 3a^2$ or $144 - 3a^2$	A1	or less simplified equiv but with at least correctly
			Attempt to find a from $\text{fg}(3) = 69$	M1	dep *M
			Obtain at least $a = 5$	A1	
			Attempt to solve $4x - 10 = x$ or $\frac{1}{4}(x + 10) = x$ or $4x - 10 = \frac{1}{4}(x + 10)$	M1	for their a; must be linear equation in one varia finding inverse of g
			Obtain $\frac{10}{3}$	A1	and no other answer
				[6]	
9	(i)		State $\cos\theta\cos 45 - \sin\theta\sin 45$	B1	or equiv including use of decimal approximatio
			Use correct identity for $\sin 2\theta$ or $\cos 2\theta$	B1	must be used; not earned for just a separate stat
			Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors involving $\sin\theta\cos\theta$
			Obtain $\sin^2\theta$	A1	AG; necessary detail needed
				[4]	
9	(ii)		Use identity to produce equation of form $\sin\frac{1}{2}\theta = c$	M1	condoning single value of constant c here (incl range -1 to 1); M0 for $\sin\theta = c$ unless value(s doubled
			Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…
			Obtain -70.5 or -70.6	A1√	or greater accuracy -70.528…; following first a answer between –90 and 90; answer(s) only : 0/3
				[3]	
9	(iii)		State or imply $6\sin^2\frac{1}{3}\theta = k$	B1	
			Attempt to relate k to at least $6\sin^2 30°$	M1	
			Obtain $0 < k < \frac{3}{2}$	A1	condone use of $\leq$
				[3]	

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